MATH 115 FINAL EXAM REVIEW NOTES

ALGEBRAIC EXPRESSIONS

***Distributive Property of* • *over* +** $a(b+c) = ab + ac$

Simplification – remove parentheses and combine **like terms**

Example

$$\begin{aligned} & 2x - 3(x+5) - (x-2) \\ &= 2x - 3x - 15 - x + 2 \\ &= 2x - 3x - x - 15 + 2 \\ &= -2x - 13 \end{aligned}$$

Exponent Rules:

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$

$$a^m \cdot a^n = a^{m+n}$$

$$\left(a^m\right)^n = a^{mn}$$

$$\left(ab\right)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0$$

$$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

<u>Examples</u>

1.
$$\begin{aligned} & x^2\left(x^{-3}\right) \\ &= x^{2+(-3)} \\ &= x^{-1} \\ &= \frac{1}{x} \end{aligned}$$

2.
$$\begin{aligned} & \frac{-2x^3y^{-2}}{7x^{-3}y^4} \\ &= \frac{-2x^3x^3}{7y^4y^2} \\ &= \frac{-2x^6}{7y^6} \end{aligned}$$

3.
$$\begin{aligned} & x^0 + 3x^0 - (-3x)^0 \\ &= 1 + 3 \cdot 1 - 1 \\ &= 3 \end{aligned}$$

4.
$$\begin{aligned} & \left(\frac{2x^2y^{-1}z^{-3}}{5xy^2z^4}\right)^{-2} \\ &= \frac{2^{-2}x^{-4}y^2z^6}{5^{-2}x^{-2}y^{-4}z^{-8}} \\ &= \frac{5^2x^2y^2y^4z^6z^8}{2^2x^4} \\ &= \frac{25x^2y^6z^{14}}{4x^4} \\ &= \frac{25y^6z^{14}}{4x^2} \end{aligned}$$

A. Polynomial Expressions

- ***Addition and Subtraction*** – combine like terms

- ***Multiplication*** – use the distributive property and applicable exponent rules
  Examples

  $$4x\left(x^2-3x-5\right)=4x^3-12x^2-20x$$

  $$(5x-1)(x+7)=5x^2+35x-x-7=5x^2+34x-7 \quad \text{(FOIL)}$$

  $$(x+5)^2=(x+5)(x+5)=x^2+10x+25$$

- ***Division*** – arrange dividend and divisor in descending order before doing **long division**
  Example

  $$\begin{array}{r} x^2+7x+8 \\ x-2\overline{)x^3+5x^2-6x+9} \\ \underline{-(x^3-2x^2)} \\ 7x^2-6x+9 \\ \underline{-(7x^2-14x)} \\ 8x+9 \\ \underline{-(8x-16)} \\ 25 \end{array}$$

  Answer: $x^2+7x+8+\dfrac{25}{x-2}$

- ***Factoring*** – the process of undoing multiplication
  - ***GCF*** or greatest common factor
    Example $12x^3y^2+20xy^3=4xy^2\left(3x^2+5y\right)$

  - ***Binomial*** factoring
    Examples x^2+y^2 prime

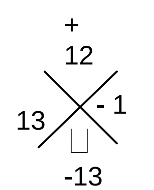
    $x^2-y^2=(x+y)(x-y)$

    $16a^2-25b^2=(4a+5b)(4a-5b)$

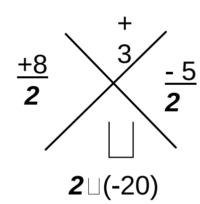
    $9t^2+16s^2$ prime

  - ***Trinomial*** factoring
    Examples

    $x^2+12x-13=(x+13)(x-1)$



    $2x^2+3x-20=(x+4)(2x-5)$



  - ***Grouping*** – 4 or more terms
    Example

    $$\begin{aligned} &x^2-3xy-2x+6y \\ &=\left(x^2-3xy\right)-\left(2x-6y\right) \\ &=x(x-3y)-2(x-3y) \\ &=(x-3y)(x-2) \end{aligned}$$

B. Rational Expressions

- ***Simplification*** – **factor** numerator and denominator and cancel common factors

  Example

$$\frac{x^2-3x+2}{x^2-4}$$
$$=\frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)}$$
$$=\frac{x-1}{x+2}$$

- ***Multiplication***: $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$

  Example

$$\frac{2x^2-x-1}{x^2-1}\cdot\frac{x^2-3x-4}{4x^2+4x+1}$$
$$=\frac{\cancel{(2x+1)}\cancel{(x-1)}(x-4)\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-1)}\cancel{(2x+1)}(2x+1)}$$
$$=\frac{x-4}{2x+1}$$

- ***Division***: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\bullet\frac{d}{c}$

  Example

$$\frac{x}{x+2}\div\frac{3x}{2x+4}$$
$$=\frac{x}{x+2}\cdot\frac{2x+4}{3x}$$
$$=\frac{\cancel{x}\cdot 2\cancel{(x+2)}}{\cancel{(x+2)}\,3\cancel{x}}$$
$$=\frac{2}{3}$$

- ***Addition and Subtraction***: combine **like fractions** (same denominator); otherwise get the **LCD** first before combining

  Examples

1.
$$\frac{x}{x+5}+\frac{x+10}{x+5}$$
$$=\frac{x+x+10}{x+5}$$
$$=\frac{2x+10}{x+5}$$
$$=\frac{2\cancel{(x+5)}}{\cancel{x+5}}$$
$$=2$$

2.
$$\frac{x}{x+5}-\frac{1}{x-3}$$
$$=\frac{x(x-3)-(x+5)}{(x+5)(x-3)}$$
$$=\frac{x^2-3x-x-5}{(x+5)(x-3)}$$
$$=\frac{x^2-4x-5}{(x+5)(x-3)}$$

Finding the LCD: Align the factors of the denominators.

Examples

1. $\dfrac{2}{3xy^2} - \dfrac{1}{9x^2y^3}$

$$\begin{aligned} 3x^3y^2 &= 3\cdot x\cdot x\cdot x \quad \cdot y\cdot y \\ 9x^2y^3 &= 3\cdot 3\cdot x\cdot x \quad \cdot y\cdot y\cdot y \\ \text{LCD} &= 3\cdot 3\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y \\ &= 9x^3y^3 \end{aligned}$$

2. $\dfrac{5}{x^2-3x+2} + \dfrac{x}{x^2-4}$

$$\begin{aligned} x^2-3x+2 &= (x-1)(x-2) \\ x^2-4 &= (x-2)(x+2) \\ \text{LCD} &= (x-1)(x-2)(x+2) \end{aligned}$$

- ***Combined Operations**:* apply **PEMDAS**

  Example

$$\begin{aligned} &\frac{x}{x+2} - \frac{x+1}{x} \cdot \frac{2x}{4x+4} \\ &= \frac{x}{x+2} - \frac{\cancel{(x+1)}\,2\cancel{x}}{\cancel{x}\cdot 4\,\cancel{(x+1)}} \\ &= \frac{x}{x+2} - \frac{1}{2} \\ &= \frac{2x-(x+2)}{2(x+2)} \\ &= \frac{2x-x-2}{2(x+2)} \\ &= \frac{x-2}{2(x+2)} \end{aligned}$$

- ***Complex Fractions***

  Examples

1.

$$\begin{aligned} &\frac{\frac{2}{x^2}}{\frac{8}{x^3}} \\ &= \frac{2}{x^2} \cdot \frac{x^3}{8} \\ &= \frac{\cancel{2}x^{3-2}}{\cancel{2}\cdot 4} \\ &= \frac{x}{4} \end{aligned}$$

2.

$$\begin{aligned} &\frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} - \frac{1}{y}} \\ &= \frac{\frac{\mathbf{y}^2}{x^2\mathbf{y}^2} - \frac{\mathbf{x}^2}{\mathbf{x}^2y^2}}{\frac{\mathbf{y}}{x\mathbf{y}} - \frac{\mathbf{x}}{\mathbf{x}y}} \\ &= \frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y-x}{xy}} \\ &= \frac{y^2-x^2}{x^2y^2} \div \frac{y-x}{xy} \\ &= \frac{y^2-x^2}{x^2y^2} \cdot \frac{xy}{y-x} \\ &= \frac{\cancel{(y-x)}(y+x)xy}{x^2y^2\,\cancel{(y-x)}} \\ &= \frac{y+x}{x^{2-1}y^{2-1}} \\ &= \frac{y+x}{xy} \end{aligned}$$

C. Radical Expressions (square roots): assume all variables are positive

- ***Simplification***

  Examples $\sqrt{25x^2y^4z^{16}} = 5xy^2z^8$

  $$\begin{aligned} &\sqrt{32x^3y^5z^6} \\ &= \sqrt{16x^2y^4z^6 \cdot 2xy} \\ &= 4xy^2z^3\sqrt{2xy} \end{aligned}$$

- ***Addition and Subtraction***: combine **like radicals**

  Examples $4\sqrt{3x} - 9\sqrt{3x} + 12\sqrt{3x} = 7\sqrt{3x}$

  $$\begin{aligned} &2y\sqrt{50x^2y} - 3x\sqrt{8y^3} \\ &= 2y\sqrt{25x^2 \cdot 2y} - 3x\sqrt{4y^2 \cdot 2y} \\ &= 2y \cdot 5x\sqrt{2y} - 3x \cdot 2y\sqrt{2y} \\ &= 10xy\sqrt{2y} - 6xy\sqrt{2y} \\ &= 4xy\sqrt{2y} \end{aligned}$$

- ***Multiplication***: multiply the radicands (must be the **same index**)

  Example

  $$\begin{aligned} &3\sqrt{2} \cdot 4\sqrt{5} \\ &= 3 \cdot 4 \cdot \sqrt{2 \cdot 5} \\ &= 12\sqrt{10} \end{aligned}$$

- ***Division***: divide the radicands (must be the **same index**)

  Example

  $$\begin{aligned} &\frac{\sqrt{72x^5y}}{\sqrt{2xy}} \\ &= \sqrt{\frac{72x^5y}{2xy}} \\ &= \sqrt{36x^4} \\ &= 6x^2 \end{aligned}$$

- ***Rationalizing the denominator*** – means removing the radical(s) from the denominator

Examples

1. $\dfrac{3}{\sqrt{7}}$

$= \dfrac{3}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}}$

$= \dfrac{3\sqrt{7}}{7}$

2. $\sqrt{\dfrac{2}{3}}$

$= \sqrt{\dfrac{2}{3} \cdot \dfrac{3}{3}}$

$= \sqrt{\dfrac{6}{9}}$

$= \dfrac{\sqrt{6}}{3}$

OR

$\sqrt{\dfrac{2}{3}}$

$= \dfrac{\sqrt{2}}{\sqrt{3}}$

$= \dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{6}}{3}$

3. $\dfrac{3}{\sqrt{2} + \sqrt{5}}$

$= \dfrac{3}{\sqrt{2} + \sqrt{5}} \cdot \dfrac{\sqrt{2} - \sqrt{5}}{\sqrt{2} - \sqrt{5}}$

$= \dfrac{3\left(\sqrt{2} - \sqrt{5}\right)}{2 - 5}$

$= \dfrac{\cancel{3}\left(\sqrt{2} - \sqrt{5}\right)}{-\cancel{3}}$

$= -\sqrt{2} + \sqrt{5}$

EQUATIONS IN ONE VARIABLE

A. Linear Equations

Strategy: Simplify each side of the equation by removing parentheses and combining like terms. Isolate the variable using the addition and/or multiplication properties of equality.

Example:

$$
\begin{aligned}
2x - 5(x + 3) - 4 &= 3(x - 1) + 5 \\
2x - 5x - 15 - 4 &= 3x - 3 + 5 \\
3x - 19 &= 3x + 2 \\
3x - 3x &= 2 + 19 \\
-6x &= 21 \\
x &= -21/6 \\
x &= -7/2
\end{aligned}
$$

B. Quadratic Equations

***Strategy**: Write the equation in the form $ax^2 + bx + c = 0$ and then either **factor and set each factor to zero** OR use the **quadratic formula** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$*

Example:

$$\begin{aligned} x^2 - 2x &= 8 \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \\ x - 4 = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = 4 \quad &\text{or} \quad x = -2 \end{aligned}$$

or

$$a = 1,\ b = -2,\ c = -8$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$$

$$x = 4 \quad \text{or} \quad x = -2$$

***Note**: **1.** For an equation of degree 3 or higher, factor first to get linear and/or quadratic factors.*
***2.** The **method of completing the square** can be used instead of the quadratic formula.*

C. Rational Equations

***Strategy**: Multiply the equation by the LCD. WATCH out for RESTRICTIONS!*
Example:

$$\frac{1}{x-1} + \frac{2}{x+1} = \frac{3}{x^2-1}; \quad LCD = (x-1)(x+1);\ \text{Restrictions: } x \neq \pm 1$$

$$(x-1)(x+1)\left[\frac{1}{x-1} + \frac{2}{x+1}\right] = (x-1)(x+1)\frac{3}{(x+1)(x-1)}$$

$$\cancel{(x-1)}(x+1)\frac{1}{\cancel{x-1}} + (x-1)\cancel{(x+1)}\frac{2}{\cancel{x+1}} = \cancel{(x-1)}\,\cancel{(x+1)}\frac{3}{\cancel{(x+1)}\,\cancel{(x-1)}}$$

$$\begin{aligned} x + 1 + 2(x-1) &= 3 \\ x + 1 + 2x - 2 &= 3 \\ 3x - 1 &= 3 \\ 3x &= 4 \\ x &= 4/3 \end{aligned}$$

D. Proportions

***Strategy:** Cross-multiply. May also be solved like a regular rational equation.*
Example:

$$\begin{aligned} \frac{2}{x+5} &= \frac{3}{x+8} \\ 2(x+8) &= 3(x+5) \\ 2x + 16 &= 3x + 15 \\ 16 - 15 &= 3x - 2x \\ 1 &= x \end{aligned}$$

E. Equations with Fractions

***Strategy**: Multiply the equation by the LCD.*
Example:

$$\frac{2}{3}(x+1) - \frac{1}{5}x = \frac{1}{2}; \quad LCD = 30$$

$$30\left[\frac{2}{3}(x+1) - \frac{1}{5}x\right] = 30\left(\frac{1}{2}\right)$$

$$\cancel{30} \cdot \frac{2}{\cancel{3}}(x+1) - \cancel{30} \cdot \frac{1}{\cancel{5}}x = \cancel{30}\left(\frac{1}{\cancel{2}}\right)$$

$$20(x+1) - 6x = 15$$

$$\vdots$$

F. Equations with Decimals

***Strategy**: Multiply the equation by a power of 10 to remove the decimal.*

Example:

$$0.5x - 0.02(x+3) = 0.01; \quad \text{Multiply by } 100$$
$$50x - 2(x+3) = 1$$
$$50x - 2x - 6 = 1$$
$$\vdots$$

G. Equations with Square Roots

*Strategy: **Isolate one radical** and square both sides.*

Example:

$$\sqrt{x+1} - 3 = 2$$
$$\sqrt{x+1} = 5$$
$$\left(\sqrt{x+1}\right)^2 = 5^2$$
$$x + 1 = 25$$
$$x = 24$$

CHECK!

SOLVING TWO LINEAR EQUATIONS IN TWO VARIABLES

- equivalent to finding the point of intersection of two lines, if any

*2 methods: **substitution** or **addition***

Example 1:

$$\begin{cases} x = y + 1 & (1) \\ 2x + 3y = 4 & (2) \end{cases}$$

substitute (1) into (2):

$$2(y+1) + 3y = 4$$
$$2y + 2 + 3y = 4$$
$$\vdots$$

Example 2:

$$\begin{cases} 3x - 2y = 1 & (1) \\ 2x + 5y = 7 & (2) \end{cases}$$

multiply (1) by 5 and (2) by 2, then add to eliminate y :

$$15x - 10y = 5$$
$$\underline{4x + 10y = 14}$$
$$19x = 19$$

$x = 1$ substitute in either equation to get y

LINES

Equations of a Line

- ***Standard form*** $\quad ax + by = c \quad$ a and b not both 0
- ***Slope-intercept form*** $\quad y = mx + b \quad$ slope m, y-intercept $(0, b)$
- ***Point-slope form*** $\quad y - y_1 = m(x - x_1) \quad$ slope m, point (x_1, y_1)
- ***Vertical line** through* (a, b) $\quad x = a$
- ***Horizontal line** through* (a, b) $\quad y = b$

Graphing Lines

- ***Table*** – ASSIGN a value to one variable and SOLVE for the other
  Examples

$y = 2x - 3$ Assign values to *x,* solve for *y*

x	*y*
0	-3
1	-1
2	1

Then plot the points $(0,-3),(1,-1),$ and $(2,1)$ and connect.

$x = y + 5$ Assign values to *y*, solve for *x*

x	*y*
5	**0**
6	**1**
4	**-1**

Then plot the points $(5,0),(6,1),$ and $(4,-1)$ and connect.

- ***Intercepts*** – get the *x*- and *y*-intercepts of the graph;
  set $x = 0$ and solve for *y* to get the *y*-intercept;
  set $y = 0$ and solve for *x* to get the *x*-intercept
  - works best when the equation is in ***standard form***

Example

$2x - 5y = -20$

x	*y*
0	4
-10	**0**

Then plot the points $(0,4)$ and $(-10,0)$ and connect.

The *x*-intercept is $(0,4)$ and the *y*-intercept is $(-10,0)$.

- ***Slope and y-intercept*** – works best when the equation is given in **slope-intercept form**
  Examples

$y = 3x + 4$

Start with the point $(0,4)$ on the *y*-axis.

The slope is $3 = \frac{3}{1} = \frac{rise}{run}$ means from $(0,4)$ go **up** 3 units and then go **right** 1 unit. The second point is $(1,7)$. Connect the two points.

$y = -\frac{2}{5}x - 3$

Start with the point $(0,-3)$ on the *y*-axis.

The slope is $-\frac{2}{5} = \frac{-2}{5} = \frac{rise}{run}$ which means from $(0,-3)$ go **down** 2 units and then go **right** 5 units to get to the point $(5,-5)$. Connect the two points.

Note: We could have written $-\frac{2}{5} = \frac{2}{-5}$, in which case we go **up** 2 units from $(0,-3)$ and then **left** 5 units to get to $(-5,-1)$. We land on the same line.

The Slope of a Line

- gives the **direction** or **steepness** of the line
- a **positive slope** means the line **rises** (or goes up) from left to right
- a **negative slope** means the line **falls** (or goes down) from left to right
- **vertical lines** have **no slope** or **undefined slope**
- **horizontal lines** have **zero** slope

3 ways to find the slope of a line:

1. given two points on the line (x_1, y_1) and (x_2, y_2),

   Strategy: use the **formula** $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y_1 - y_2}{x_1 - x_2}$

   Example $(1,6)$ and $(-3,4)$; $m = \frac{4-6}{-3-1} = \frac{-2}{-4} = \frac{1}{2}$

2. given the graph,
   Strategy: **count** how you go up/down and left/right to go from one point to the next

3. given the equation,
   Strategy: make sure the equation is in **slope-intercept** form, i.e., ***y* is isolated**;
   the coefficient of *x* is the slope

   Examples $y = 5x - 9 \Rightarrow m = 5$

   $x = y - 3 \Rightarrow y = x + 3 \Rightarrow m = 1$

   $2x + 3y = 5 \Rightarrow 3y = -2x + 5 \Rightarrow y = -\frac{2}{3}x + \frac{5}{3} \Rightarrow m = -\frac{2}{3}$

Parallel and Perpendicular Lines

Parallel lines have the ***same slope.***
Perpendicular lines have **slopes** whose **product is -1**, i.e. the slope of one line is the **negative (opposite) reciprocal** of the slope of the other.

Examples $y = 3x - 5$ and $y = 3x - 23$ are parallel since $m = 3$ for both

$y = 3x - 5$ and $y = -\frac{1}{3}x + 17$ are perpendicular since $(3)\left(-\frac{1}{3}\right) = -1$

Vertical and Horizontal Lines

A vertical line is **parallel to the *y*-axis** and **perpendicular to the *x*-axis**.
A horizontal line is **parallel to the *x*-axis** and **perpendicular to the *y*-axis**.

Writing Equations of Lines

Necessary Ingredients: **A point** and **THE slope**
Baking Pan: *the **point-slope** form* $y - y_1 = m(x - x_1)$

*Strategy for **non-vertical** and **non-horizontal** lines:*

a. *Check if the 2 ingredients are given. If yes, go to step c, otherwise, go to step b.*

b. *Solve for the missing ingredient. Then go to Step c.*

c. *Plug in the given values of the slope **m** and the **x_1** and **y_1** of the point in the **point-slope** form of the equation of a line.* $y - y_1 = m(x - x_1)$

d. *"Clean up" your answer in Step c to get either the **standard** form or **slope-intercept** form.*

Examples

1. line contains $(1,2)$ and has slope 7
   Both ingredients are given so start with Step c:
   $y - 2 = 7(x - 1)$
   Write in standard form:
   $y - 2 = 7x - 7$
   $y = 7x - 5$
   $-7x + y = -5$

2. line goes through $(1,2)$ and $(3,4)$

   The slope is **not given** so we need to solve for it and we can use the formula:
   $m = \frac{4-2}{3-1} = \frac{2}{2} = 1$. Then go to Step c using either point $(1,2)$ or $(3,4)$. If we use $(1,2)$, we get

$$y - 2 = 1(x-1)$$
$$y - 2 = x - 1$$
$$y = x + 1$$
$$-x + y = 1$$

3. line goes through $(1,2)$ and is parallel to $6x - y = 5$

   The slope is **not given** but we know the line we want is parallel to the given line.
   From the given equation,

$$6x - y = 5 \Rightarrow -y = -6x + 5 \Rightarrow y = 6x - 5 \Rightarrow m = 6$$

   and since the line we want is **parallel** to this line then the slope is **the same**.
   Thus,

$$y - 2 = 6(x-1)$$
$$y - 2 = 6x - 6$$
$$y = 6x - 4$$
$$-6x + y = -4$$

4. line contains $(1,2)$ and is perpendicular to $2x + 5y = 9$

   The slope is **not given** but we know the line we want is perpendicular to the given line. From the given equation,

$$2x + 5y = 9 \Rightarrow 5y = -2x + 9 \Rightarrow y = -\frac{2}{5}x + \frac{9}{5} \Rightarrow m = -\frac{2}{5}$$

   and since the line we want is **perpendicular** to this line then the slope is
   the **negative reciprocal** which is $+\frac{5}{2}$. Thus,

$$y - 2 = \frac{5}{2}(x-1)$$
$$2(y-2) = 5(x-1)$$
$$2y - 4 = 5x - 5$$
$$2y = 5x - 1$$
$$-5x + 2y = -1$$

5. line contains the *x*-intercept of $4x + 3y = 12$ and has slope -5

   The slope is given but **no point** is given. The problem says use the
   x-intercept of the given line. Setting $y = 0$, we get $4x = 12$ or $x = 3$,
   and the point we can use is $(3,0)$. Thus,

$$y - 0 = -5(x-3)$$
$$y = -5x + 15$$
$$5x + y = 15$$

*Strategy for **vertical** and **horizontal** lines:* **SKETCH!**

Examples
- vertical line through $(1,2)$ is the line $x = 1$
- horizontal line through $(1,2)$ is the line $y = 2$
- line through $(1,2)$ that is parallel to the *x*-axis is the line $y = 2$
- line through $(1,2)$ that is perpendicular to the *x*-axis is the line $x = 1$